Statistics

Fall 2022

Lecture 10

Feb 19-8:47 AM

Class QZ 10

Consider a Poisson prob. dist. with $\mu = 8$.

1) find $P(x=10) = \text{poissonpdf}(8,10) = \boxed{.099}$ ✓

2) find $P(x<15) = P(x \leq 14) = \text{poissoncdf}(8,14)$
$= \boxed{.983}$ ✓

Oct 25-9:09 PM

Clear all lists.

Store 4, 8, 12, 16 in L1

Use 1-Var Stats with L1 to find

$\boxed{\mu = 10}$ $\sigma = 4.472$ $\boxed{\sigma^2(\text{exact}) = 20}$

Take all samples of $\boxed{\text{size } 2}$ with replacement from this data.

4,4	4,8	4,12	4,16
8,4	8,8	8,12	8,16
12,4	12,8	12,12	12,16
16,4	16,8	16,12	16,16

Now find $\bar{x}$ of each sample

4	6	8	10
6	8	10	12
8	10	12	14
10	12	14	16

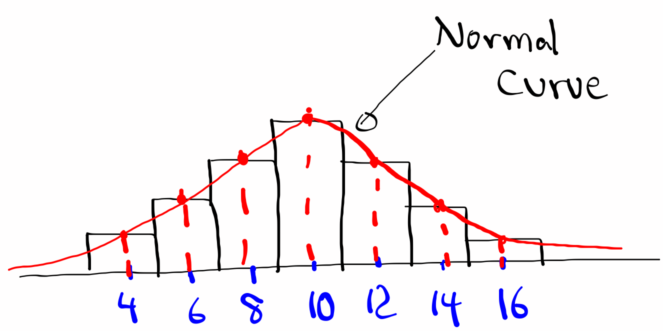
16 Means

$\bar{x}$	$P(\bar{x})$
4	1/16
6	2/16
8	3/16
10	4/16
12	3/16
14	2/16
16	1/16

Nov 1-6:54 PM

$\bar{x}$	$P(\bar{x})$
4	1/16
6	2/16
8	3/16
10	4/16
12	3/16
14	2/16
16	1/16

Draw Prob. dist. histogram



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Use 1-Var Stats with L2 & L3

To find

$\boxed{\mu_{\bar{x}} = 10}$ $\sigma_{\bar{x}} = 3.162$ $\boxed{\sigma^2_{\bar{x}}(\text{exact}) = 10}$

Nov 1-7:01 PM

Central - Limit Theorem

$\boxed{\mu_{\bar{x}} = \mu}$

$\sigma^2_{\bar{x}} = \dfrac{\sigma^2}{n}$

$\boxed{\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}}$

Suppose a normal Prob. dist $\mu = 85$, & $\sigma = 10$

we take all Samples of Size 4.

$\mu_{\bar{x}} = \mu = 85$

$\sigma^2_{\bar{x}} = \dfrac{\sigma^2}{n} = \dfrac{10^2}{4} = 25$

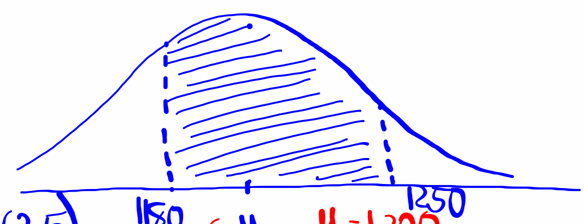
$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{4}} = \dfrac{10}{2} = 5$

Oct 25-9:04 PM

Suppose SAT Scores have a normal dist with $\mu = 1200$ and $\sigma = 125$. $N(1200, 125)$

If we take Samples of $\boxed{\text{Size } 4}$ ($n=4$), find the Prob. that $\boxed{\text{their mean}}$ ($\bar{x}$) Score is between 1180 and 1250.

$P(1180 < \bar{x} < 1250)$



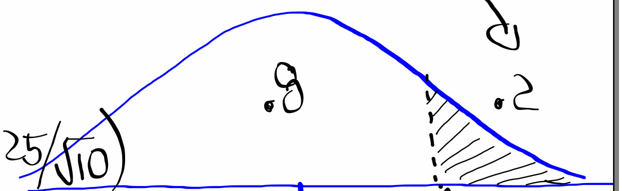
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 1200 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{125}{\sqrt{4}} = 62.5 \end{cases}$

$= \text{normalcdf}(1180, 1250, 1200, 62.5)$

$= \boxed{.414}$

Nov 1-7:09 PM

For randomly selected groups of 10 SAT exams, find the mean that separates the top 20% from the rest.



$\bar{x} = \text{invNorm}(.8, 1200, 125/\sqrt{10})$

$= 1233.268$

$\approx \boxed{1233}$

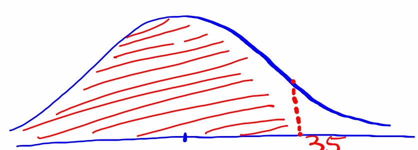
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 1200 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{125}{\sqrt{10}} \end{cases}$ $\bar{x}$

Nov 1-7:15 PM

Ages of all students are normally distributed with mean of 30.5 Yrs and standard deviation of 5.8 Yrs. $N(30.5, 5.8)$

If we randomly select $\boxed{\text{groups of } \underset{n=5}{5}}$ students, find the prob. that $\boxed{\text{their mean}}_{\bar{x}}$ age is $\boxed{\text{below } 35}$ Yrs.

$P(\bar{x} < 35)$

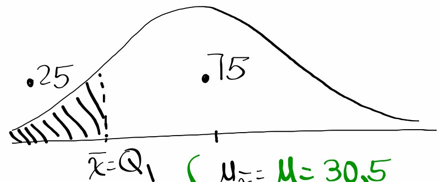


$= \text{normalcdf}(-E99, 35, 30.5, 5.8/\sqrt{5})$ CLT $\begin{cases} \mu_{\bar{x}} = \mu = 30.5 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{5.8}{\sqrt{5}} \end{cases}$

$= \boxed{.959} = 95.9\% \approx 96\%$

Nov 1-7:20 PM

Find $\bar{x} = Q_1$, Round to 1-decimal, for randomly selected groups of 4 students.



$$\text{CLT}\begin{cases}\mu_{\bar{x}} = \mu = 30.5 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{5.8}{\sqrt{4}} = 2.9\end{cases}$$

$\bar{x} = \text{invNorm}(.25, 30.5, 2.9)$

$= 28.544 \approx \boxed{28.5}$

SG 20 & SG 21 ✓

Exam 2 is next week
SG 1 – SG 21

Nov 1-7:26 PM

$Z_{\alpha/2}$ $\quad \alpha \to$ Alpha

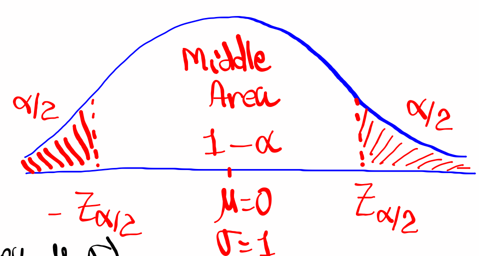
$0 < \alpha < 1$

$\alpha \to$ significance level

$(1-\alpha)\cdot 100\% \to$ Confidence level

$1-\alpha$ is the middle area of graph of dist.

$Z_{\alpha/2}$ is the value that separates the top $\alpha/2$ area from the rest.



How to find $Z_{\alpha/2}$:

invNorm(Left area, μ, σ)

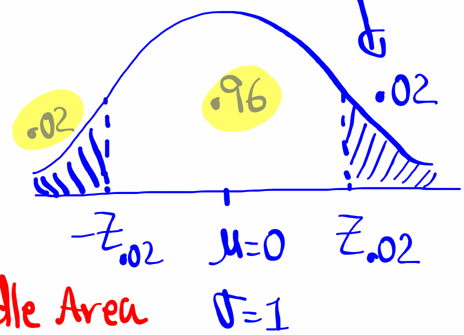
Nov 1-7:45 PM

Find $Z_{.02}$

Right-Tail Area

$\frac{\alpha}{2} = .02 \rightarrow \alpha = .04$

$1-\alpha = 1 - .04 = .96$ Middle Area



$.96 = 96\%$ Confidence level

$Z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

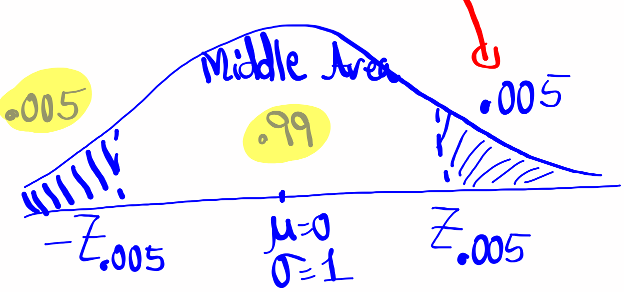
Nov 1-7:49 PM

Find $Z_{\alpha/2}$ for $\alpha = .01$ $\quad 1-\alpha = .99$

$\alpha/2 = .01/2 = .005$

Find $Z_{.005}$

Right-area



$Z_{.005} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

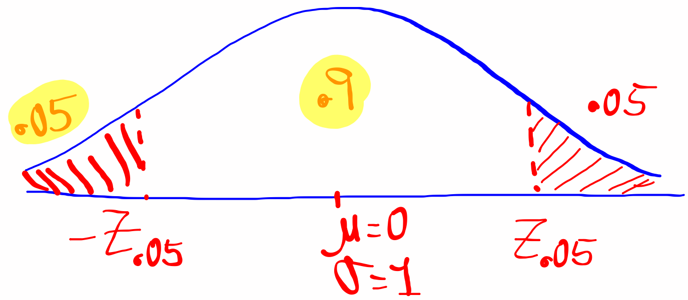
Nov 1-7:53 PM

Find $Z_{\alpha/2}$ for $\underbrace{\text{90\% Confidence level.}}$

Middle Area .9

$1-.9=.1 \leftarrow \alpha$

$.1/2=.05 \leftarrow \alpha/2$

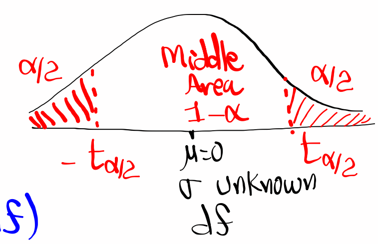


$Z_{.05}=\text{invNorm}(.95, 0, 1)=\boxed{1.645}$

Nov 1-7:56 PM

t-Dist

- It is bell-shape, symmetric
- Total area = 1
- $\mu=0$, but σ unknown
- It comes with degrees of freedom



How to find $t_{\alpha/2}$:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

invT(Left area, df)

If you don't have invT,

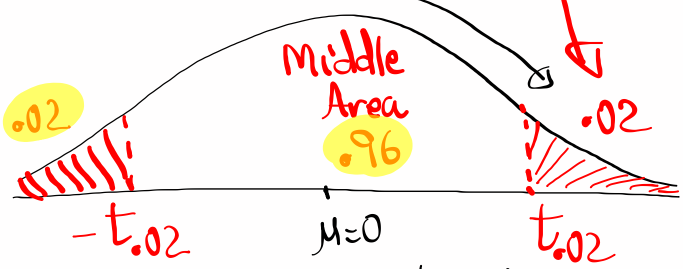
Time to download recommended App in the Syllabus

Try G.Calculator

Nov 1-7:59 PM

find $t_{.02}$ with $df = 9.$

↳ Right-Tail Area



σ unknown

$df = 9$

$t_{.02} = \text{invT}(.98, 9) = \boxed{2.398}$

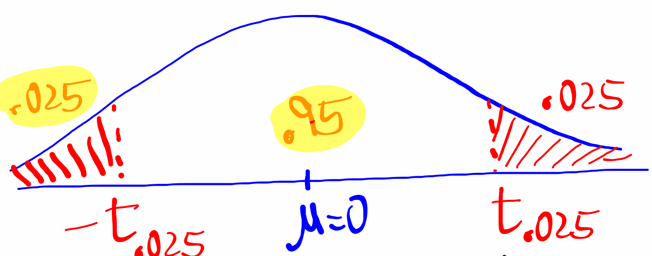
Nov 1-8:04 PM

find $t_{\alpha/2}$ for $\alpha = .05$ with $df = 19.$

$1 - \alpha = .95$

$\alpha/2 = .05/2 = .025$

find $t_{.025}$



σ unknown

$df = 19$

$t_{.025} = \text{invT}(.975, 19)$

$= \boxed{2.093}$

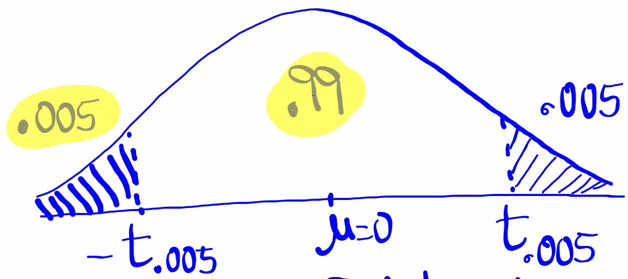
Nov 1-8:08 PM

Find $t_{\alpha/2}$ for 99% conf. level with df=15.

Middle Area
.99

$1 - .99 = .01$

$.01 \div 2 = .005$



σ Unknown

df=15

$t_{.005} = \text{invT}(.995, 15)$

$= \boxed{2.947}$

Nov 1-8:10 PM

What is degrees of freedom?

18 Students, 18 Donuts

First person → 18 choices
Second " → 17 "
Third " → 16 "
⋮
Last person → (No choice) 1 donut left

df = 17

You have 7 clean shirts.

Monday → 7 choices
Tuesday → 6 "
Wednesday → 5 "
⋮
Sunday → No choice (1 clean shirt)

df = 6

Nov 1-8:14 PM

(SG 22 & 23)

Estimating Parameters:

Parameters such as

- Population Proportion P
- Population Mean μ
- Population standard deviation σ

Nov 1-8:30 PM

Our estimation will be $\underbrace{\text{range of values}}_{\text{Confidence Interval}}$

Every confidence Interval comes with

confidence level.

$(1-\alpha)\cdot 100\%$

when conf. level not given,

$\Rightarrow$ use 95% c-level.

Nov 1-8:32 PM

Confidence Interval for Population Proportion P:

Final Answer → $\quad < P <$

Format → $\hat{p} - E < P < \hat{p} + E$

P-hat
Sample Proportion
Point-estimate

Margin of error

$\hat{p} = \dfrac{x}{n}$ ← # of favorable resp. ← Sample Size

$x = n\hat{p}$ if decimal → Round-up

$\hat{p} + \hat{q} = 1$, $\hat{q} = 1 - \hat{p}$

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$Z_{\alpha/2}$ is for $(1-\alpha)\cdot 100\%$ C-level.

Nov 1-8:35 PM

In a survey of 250 voters, 90 of them were in favor of certain candidate for mayor election.

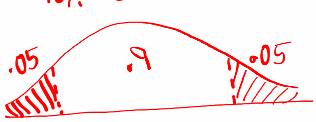
$n = 250$, $x = 90$

$\hat{p} = \dfrac{x}{n} = \dfrac{90}{250} = .36$

$\hat{q} = 1 - \hat{p} = .64$

Find 90% confidence interval for the prop. of all voters in support of that candidate.

90% C-level



$E = Z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

$= 1.645 \cdot \sqrt{\dfrac{(.36)(.64)}{250}}$

$= .050$ $\quad \boxed{E = .05}$

$Z_{.05} = \text{invNorm}(.95, 0, 1) = 1.645$

$\hat{p} - E < P < \hat{p} + E$

$.36 - .05 < P < .36 + .05$

$\boxed{.31 < P < .41}$

Among all voters, between 31% & 41% support that candidate.

Nov 1-8:40 PM

Now Using TI:

$\boxed{\text{STAT}}$ $\boxed{\rightarrow}$ TESTS

$\boxed{\text{1-PropZInt}}$ → x: 90

n: 250

C-level: .9

$\boxed{\text{Calculate}}$

$\boxed{.310 < p < .410}$

$E = \dfrac{.410 - .310}{2} = \boxed{.05}$

$\hat{p} = \dfrac{.410 + .310}{2} = \boxed{.36}$

Nov 1-8:48 PM

In a Sample of 400 students, 8.5% of them were Smokers.

$n = 400$

$\hat{p} = .085$ $\Rightarrow x = n\hat{p} = 400(.085)$

$x = 34$

Find 99% Conf. interval for the prop. of all students that are Smokers.

1-Prop Z Int

$x = 34$

$n = 400$

C-level = .99

$\boxed{.049 < p < .121}$

we are 99% confident that between 5% and 12% of all students Smoke.

$E = \dfrac{.121 - .049}{2} = \boxed{.036}$

$\hat{p} = \dfrac{.121 + .049}{2} = \boxed{.085}$

Nov 1-8:52 PM

Confidence Interval for population mean μ:

Final Answer : $\quad < \mu <$

Format : $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample Mean, Point-estimate

E: Margin of error

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ → $df = n-1$
TI Command: ZInterval	TI Command: TInterval
inpt: $\boxed{\text{Stats}}$	Inpt: $\boxed{\text{Stats}}$

$E = \frac{\quad - \quad}{2}$ $\qquad \bar{x} = \frac{\quad + \quad}{2}$

Nov 1-8:58 PM

Given : $n = 32$, $\bar{x} = 88$, $\sigma = 12$, C-level: .98

Find confidence interval for μ

Since σ Known ⟹ Z Interval

Inpt: $\boxed{\text{STATS}}$

σ: 12

$\bar{x}$: 88

$n = 32$

C-level: .98

$\boxed{\text{Calculate}}$

$83.065 < \mu < 92.935$

$83 < \mu < 93$

$E = \frac{93 - 83}{2} = 5$

$\bar{x} = \frac{93 + 83}{2} = 88$

Nov 1-9:04 PM

Given $n=15$, $\bar{x}=125$, $S=18$, C-level: .9

Find Conf. interval for μ.

σ unknown $\rightarrow$ TInterval

inpt: $\boxed{\text{Stats}}$

$\bar{x}=125$

$S=18$

$n=15$

C-level: .9

$\boxed{\text{calculate}}$

$116.81 < \mu < 133.19$

$\boxed{117 < \mu < 133}$

$E=\frac{133-117}{2}=\boxed{8}$

$\bar{x}=\frac{133+117}{2}=\boxed{125}$

Nov 1-9:08 PM

Class QZ 11

Consider a geometric Prob. dist with $p=.5$

1) $P(x=3)=\text{geometpdf}(.5,3)=\boxed{.125}$

2) $P(x>3)=P(x\geq 4)=1-P(x\leq 3)$

$=1-\text{geometcdf}(.5,3)=\boxed{.125}$

Nov 1-9:13 PM